Demystifying Multilingual Chain-of-Thought in Process Reward Modeling

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Abstract

Large language models (LLMs) are designed to perform a wide range of tasks. To improve their ability to solve complex problems requiring multi-step reasoning, recent research leverages process reward modeling to provide fine-grained feedback at each step of the reasoning process for reinforcement learning (RL), but it predominantly focuses on English. In this paper, we tackle the critical challenge of extending process reward models (PRMs) to multilingual settings. To achieve this, we train multilingual PRMs on a dataset spanning seven languages, which is translated from English. Through comprehensive evaluations on two widely used reasoning benchmarks across 11 languages, we demonstrate that multilingual PRMs not only improve average accuracy but also reduce early-stage reasoning errors. Furthermore, our results highlight the sensitivity of multilingual PRMs to both the number of training languages and the volume of English data, while also uncovering the benefits arising from more candidate responses and trainable parameters. This work opens promising avenues for robust multilingual applications in complex, multi-step reasoning tasks. In addition, we release the code to foster research along this line.[1]

1 Introduction

Aligning large language models (LLMs) with human preferences can significantly improve the model performance across various downstream tasks (Christiano et al., 2017; Ziegler et al., 2019). This requires a reward model that is trained on human preference data (Ziegler et al., 2019; Stiennon et al., 2020; Shen et al., 2021; Ouyang et al., 2022). Typically, reward models are trained based on the final outcome of the LLMs' response, and we refer to these as outcome reward models (ORMs) (Cobbe et al., 2021a; Uesato et al., 2022; Yu et al., 2023a). However, most of recent work demonstrates that ORMs fall short on complex multi-step reasoning tasks (Uesato et al., 2022; Shao et al., 2024). To overcome this limitation, process reward models (PRMs) are introduced, providing fine-grained rewards at each step of the LLMs' chain-of-thought (Lightman et al., 2024; Li et al., 2023; Wang et al., 2024b; Ma et al., 2023). Previous research has shown that LLMs supervised by PRMs can effectively produce better responses (Wang et al., 2024b; Shao et al., 2024).

Despite these significant advances, recent research on ORMs and PRMs has predominantly focused on monolingual settings, particularly English (Lightman et al., 2024; Wang et al., 2024a,b). However, the exploration of multilingual PRMs remains relatively limited. Therefore, with the advent of multilingual LLMs, a natural research question arises: *How can we effectively train multilingual PRMs for complex, multi-step reasoning tasks?*

To address this research question, we translate the existing PRM datasets, PRM800K (Lightman et al., 2024) and Math-Shepherd (Wang et al., 2024b), from English into six additional languages, resulting in a total of seven seen languages for training. We then train multilingual PRMs using the collection of these translated datasets. We define three PRM setups: PRM-MONO, PRM-CROSS, and PRM-MULTI. The PRM-MONO setup is trained and evaluated solely on a single language, the PRM-CROSS setup is trained on one language but evaluated on all test languages, and the PRM-MULTI setup is trained on seven seen languages and evaluated on all test languages. Finally, we conduct a comprehensive evaluation on two popular reasoning tasks (MATH500 and MGSM) across 11 languages (seven seen languages and four unseen languages) using three LLMs (METAMATH-MISTRAL-7B, LLAMA-3.1-8B-MATH, and DEEPSEEKMATH-7B-INSTRUCT).

[1]https://github.com/weixuan-wang123/Multilingual-PRM

In this work, our main takeaways can be summarized as follows:

- **Multilingual PRM consistently outperforms monolingual and cross-lingual PRMs across all three LLMs.** Our results demonstrate that PRM-MULTI significantly improves model performance, boosting average accuracy by up to +1.2 and +1.5 points compared to PRM-CROSS and PRM-MONO, respectively (see Section 5.1).
- **Multilingual PRM is sensitive to both the number of languages and the amount of English training data.** Our experiment shows that training an optimal multilingual PRM requires careful consideration of how many languages to include (see Section 5.2) and how much English data to use (see Section 5.3).
- **Multilingual PRM produces fewer errors in the early steps.** We identify the first occurrences of wrong predictions made by PRMs and observe that PRM-MULTI produces fewer errors in the early steps compared to PRM-MONO and PRM-CROSS (see Section 6.1).
- **Multilingual PRM can benefit even more from more candidate responses and trainable parameters.** Our analysis demonstrates that PRM-MULTI becomes even more advantageous when there is a larger number of candidate responses (see Section 6.2) and when more trainable parameters are introduced (see Section 6.3).

2 Related Work

Reward Model in Mathematical Reasoning To advance the accuracy of mathematical reasoning, reward models (RMs) have emerged as powerful tools for evaluating and guiding solution generation. In particular, two principal RM paradigms have garnered significant attention: the Outcome Reward Models (ORMs) (Cobbe et al., 2021a; Yu et al., 2023a) and the Process Reward Models (PRMs) (Uesato et al., 2022; Lightman et al., 2024; Li et al., 2023; Ma et al., 2023; Wang et al., 2024b; Luo et al., 2024; Gao et al., 2024; Wang et al., 2024a). ORMs assign a single score to an entire solution and thereby focuses on final correctness, whereas PRMs score each individual step of the reasoning process, offering more finer-grained evaluations. As a result, PRMs provide more detailed guidance and have demonstrated greater potential in enhancing reasoning capabilities compared to ORMs (Lightman et al., 2024; Wu et al., 2023).

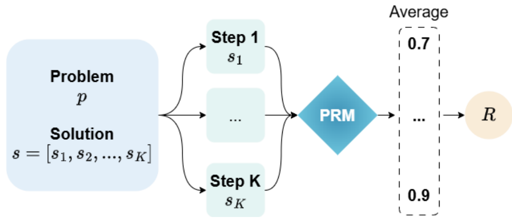

Figure 1: Framework of PRM.

Multilingual Reward Model Beyond English-language tasks, the integration of RMs into multilingual scenarios is still under-explored. Reinforcement learning approaches often rely on RMs predominantly trained on English data (Shao et al., 2024; Yang et al., 2024a). This over-representation introduces biases, as these RMs may overfit to English-specific syntactic and semantic patterns, limiting their effectiveness in cross-lingual tasks and motivating the development of multilingual RMs (Hong et al., 2024). While there is growing evidence that cross-lingual transfer is feasible (Wu et al., 2024a; Hong et al., 2024), existing research often overlooks the unique challenges of multilingual reasoning. After the release of the OpenAI-o1 model (OpenAI, 2024), PRMs, with their capability for fine-grained feedback, have attracted even greater interest. Yet, the performance of multilingual PRMs in diverse linguistic contexts remains insufficiently investigated (Yang et al., 2024b). To bridge this gap, we investigate how multilingual PRMs contribute to solving mathematical tasks across different languages, aiming to provide insights into how fine-grained process supervision can enhance reasoning capabilities beyond English, thereby contributing to the development of more universally applicable reasoning models.

3 Process Reward Modeling

3.1 PRM Training

Given a question p and its solution s, the ORM assigns a single value to s to indicate whether s is correct. We stack a binary classifier on top of the LLM and train the ORM with the binary cross-entropy loss:

$$\mathcal{L}_{\text{ORM}} = - (y_s \log(r_s) + (1 - y_s) \log(1 - r_s)) \quad (1)$$

where y_s is the ground truth label for the solution s ($y_s = 1$ if s is correct, otherwise $y_s = 0$), and r_s is the probability score that s is correct.

In contrast, the PRM evaluates each reasoning step of the solution s. The PRM is trained using the following loss function:

$$\mathcal{L}_{\text{PRM}} = -\sum_{i=1}^{K} y_{s_i} \log(r_{s_i}) + (1 - y_{s_i}) \log(1 - r_{s_i}) \quad (2)$$

where s_i is the i-th step of the solution s, y_{s_i} is the ground truth label for step s_i, r_{s_i} is the score assigned to s_i by the PRM, and K is the total number of reasoning steps in the solution s. Compared to ORM, PRM provides more detailed and reliable feedback by evaluating individual steps.

3.2 Ranking for Verification

Following Wu et al. (2024b); Lightman et al. (2024); Wang et al. (2024b), we evaluate the performance of PRM using the *best-of-N* selection evaluation paradigm (Charniak and Johnson, 2005; Cobbe et al., 2021b). Specifically, given a question, multiple solutions are sampled from an LLM (referred to as the *generator*) and re-ranked using a reward model (referred to as the *verifier*). For each solution, as shown in Figure 1, PRM assesses the correctness of each reasoning step. The scores for all steps are averaged to compute an overall score for the solution. The highest-scoring solution is then selected as the final output. This approach enhances the likelihood of selecting solutions containing correct answers, thereby improving the success rate of solving mathematical problems with LLMs.

3.3 Reinforcement Learning with Process Supervision

Using the trained PRM, we fine-tune LLMs with Policy Optimization (PPO) (Schulman et al., 2017) in a step-by-step manner. This method differs from the conventional strategy that uses PPO with an ORM, that only gives a reward at the end of the response. Conversely, our step-by-step PPO offers rewards at the end of each reasoning step.

While we analyse PRM both intrinsically (using best-of-N), and extrinsically (using PPO), we focus on best-of-N for a clean testbed without confounders from reinforcement learning.

4 Experimental Setups

Training Datasets We combine the PRM800K (Lightman et al., 2024) and Math-Shepherd (Wang et al., 2024b) as training data to finetune PRMs, and translate the combined dataset from English (en) to six languages: German (de), Spanish (es), French (fr), Russian (ru), Swahili (sw), and Chinese (zh) with using NLLB 3.3B (Costa-jussà et al., 2022). The reasoning step statistics are presented in Table 4 (Appendix A), and the parallel examples across seven languages have the same number of reasoning steps.

Test Dataset We evaluate the performance of LLMs using two widely used math reasoning datasets, `MGSM` (Shi et al., 2022) and `MATH500` (Wang et al., 2024b). For the `MATH500` datset, we translate it from English to ten languages: Bengali (bn), German (de), Spanish (es), French (fr), Japanese (ja), Russian (ru), Swahili (sw), Telugu (te), Thai (th), and Chinese (zh) with Google Translate, which is consistent with the languages included in the `MGSM` dataset. Furthermore, we also categorize the languages involved in the downstream tasks into two groups based on the training data of PRM: *seen languages* (en, de, es, fr, ru, sw, and zh) and *unseen languages* (bn, ja, te, and th).

Multilingual PRM Setups To better understand PRMs in the context of multilingual research, we define three setups: PRM-MONO, PRM-CROSS, and PRM-MULTI. The PRM-MONO setup is trained and evaluated on the same single language, serving as the baseline for monolingual PRMs. The PRM-CROSS setup is trained on one language but evaluated on all 11 test languages. Specifically, in this work, we train PRM-CROSS on the English PRM dataset unless otherwise specified. Finally, the PRM-MULTI setup represents the multilingual PRM, which is both trained on all the seen languages and evaluated on all 11 test languages. To enhance the reliability and generalizability of our study, we train our multilingual PRM (***verifier***) based on the QWEN2.5-MATH-7B-INSTRUCT (Yang et al., 2024a), and leverage three diverse LLMs as the ***generator***: METAMATH-MISTRAL-7B (Yu et al., 2023b), LLAMA-3.1-8B-MATH (fine-tuned with the MetaMath dataset (Dubey et al., 2024)),[2] and DEEPSEEKMATH-7B-INSTRUCT (Shao et al., 2024). The details of training these PRMs are presented in Appendix B.

[2] `https://huggingface.co/gohsyi/Meta-Llama-3.1-8B-sft-metamath`

MATH500	μ_{ALL}	μ_{SEEN}	μ_{UNSEEN}	en	de	es	fr	ru	sw	zh	ja	bn	te	th
							METAMATH-MISTRAL-7B							
PRM-MONO	-	42.5	-	49.0	44.4	45.8	45.6	46.0	25.0	41.8	-	-	-	-
PRM-CROSS	39.4	43.1	39.1	49.0	45.4	45.0	**46.8**	**46.4**	25.2	**43.8**	43.6	31.4	**22.0**	34.6
PRM-MULTI	**39.6**	**43.1**	**39.4**	**50.2**	**45.6**	**47.4**	45.4	45.2	**25.2**	42.8	**43.6**	**32.6**	21.8	**35.2**
							LLAMA-3.1-8B-MATH							
PRM-MONO	-	43.3	-	49.0	46.2	45.8	44.2	45.8	26.2	46.2	-	-	-	-
PRM-CROSS	40.9	43.6	36.3	49.0	48.8	**46.6**	44.8	44.8	26.0	45.2	**43.0**	**36.0**	28.2	37.8
PRM-MULTI	**41.7**	**44.8**	**36.4**	**51.0**	**48.8**	45.8	**46.0**	**46.2**	**28.4**	**47.2**	42.0	34.6	**30.2**	**38.6**
							DEEPSEEKMATH-7B-INSTRUCT							
PRM-MONO	-	55.1	-	63.0	59.0	**60.4**	59.0	60.2	29.2	**55.0**	-	-	-	-
PRM-CROSS	50.2	54.9	41.9	62.4	**60.0**	59.8	**61.4**	57.4	29.4	54.0	54.4	**38.2**	32.4	42.6
PRM-MULTI	**51.3**	**55.6**	**43.7**	**63.8**	58.6	60.2	60.2	**61.4**	**30.6**	54.2	**55.8**	38.0	**35.6**	**45.4**

Table 1: Different PRMs' best-of-N sampling (N = 64) performance on MATH500 with the generator of METAMATH-MISTRAL-7B, LLAMA-3.1-8B-MATH, and DEEPSEEKMATH-7B-INSTRUCT. μ_{ALL}, μ_{SEEN}, and μ_{UNSEEN} indicate the macro-average of results across all the languages, the seen languages, and the unseen languages, respectively.

5 Recipes for Multilingual PRM Training

In this section, we conduct a series of experiments to investigate the performance of multilingual PRM. We examine how PRM-MULTI compares to PRM-MONO and PRM-CROSS (Section 5.1), the impact of the number of training languages (Section 5.2), and the effect of varying the proportion of English in the training data (Section 5.3).

5.1 Monolingual, Cross-lingual, or Multilingual PRMs?

Building upon the findings of Wu et al. (2024b), who demonstrated that cross-lingual ORMs outperform monolingual ones, we investigate the impact of multilingualism on PRMs. Specifically, we compare PRM-MONO, PRM-CROSS, and PRM-MULTI to determine which setup offers best performance across languages.

Setup We include three setups in this work. The PRM-MONO is trained and evaluated on each individual language from the set of seen languages. The PRM-CROSS is trained exclusively on an English dataset and evaluated on all 11 test languages. Finally, the PRM-MULTI is trained on all seen languages and tested on all 11 test languages.

Multilingual PRMs perform best, followed by cross-lingual PRMs, while monolingual PRMs achieve the worst performance, on the seen languages. As shown in Table 1, PRM-MULTI consistently achieves the highest performance across multiple language generators on the seen languages, surpassing PRM-MONO and PRM-CROSS by +1.5 and +1.2 with LLAMA-3.1-8B-MATH generator, respectively. This indicates that incorporating data from multiple languages for PRM training significantly enhances the model's ability across different languages. When comparing PRM-MONO and PRM-CROSS, we observe that PRM-CROSS outperforms the PRM-MONO for the English-centric METAMATH-MISTRAL-7B and LLAMA-3.1-8B-MATH generators. We hypothesize that this advantage stems from the pre-training phase: these generators are predominantly trained on English data but have limited exposure to multilingual corpora. As a result, fine-tuning on English PRM data enhances the reasoning capabilities of PRMs, facilitating greater cross-lingual transfer. More monolingual results are in Appendix C.

Multilingual PRMs generalize better on the unseen languages. Both PRM-CROSS and PRM-MULTI are evaluated on four additional unseen languages. As shown in Table 1, PRM-MULTI demonstrates superior overall performance on the unseen languages in terms of μ_{UNSEEN}. These results suggest that training PRMs on multilingual datasets can effectively enhance model generalization to the unseen languages.

In conclusion, these findings demonstrate that training a single multilingual PRM is an effective strategy for broad cross-lingual coverage, outperforming models trained either on a target language or on English alone. This outcome supports that PRM-MULTI is particularly advantageous for expanding the capabilities of PRMs in multilingual settings. More results on MGSM are in Appendix D.

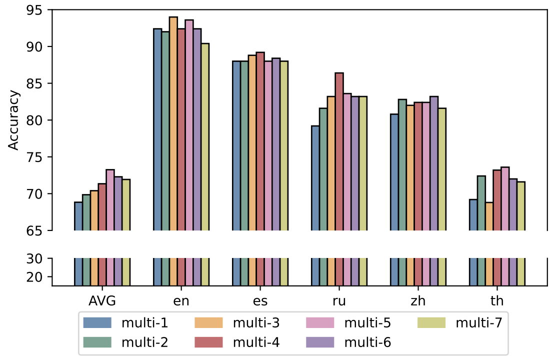

Figure 2: Best-of-N Performance on MGSM of PRMs trained using various subsets of English, German, Spanish, French, Russian, Swahili, and Chinese, with the generator of LLAMA-3.1-8B-MATH. The averages scores across all 11 languages.

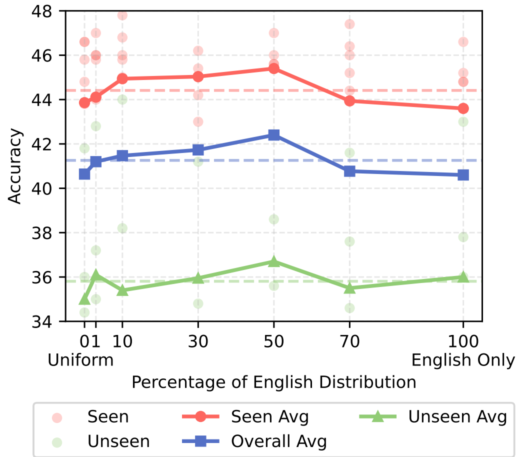

Figure 3: Best-of-N sampling performance of LLAMA-3.1-8B-MATH with PRMs finetuned on a training set where P% of the data is in English and (100 - P)% is uniformly distributed across six other languages. Each tick on the X-axis represents a specific tuning set configuration. The dash lines in blue, red, and green, indicate the average scores of all the languages, the seen languages, and the unseen languages, respectively.

5.2 Does More Languages Lead to Better Multilingual PRMs?

While multilingual PRMs have demonstrated significant improvements, the question of how many languages are needed to achieve the best performance remains an open research problem. In this section, we address this research question by exploring the relationship between the number of training languages and the resulting performance.

Setup We conduct experiments by training PRMs on datasets ranging from a single language up to all seven languages. In this section, the number of total training examples of all PRMs are fixed. When the number of languages exceeds one, the total training examples are evenly distributed across all the selected languages. For evaluation, we test all PRMs on 11 different languages. The evaluation scores are averaged for each test language across all PRMs trained with the same number of languages.

More languages do not result in better multilingual PRMs. As shown in Figure 2, the overall performance (AVG) improves as the number of training languages increases up to five languages. Beyond this point, adding more languages does not lead to further gains. Additionally, results from five individual languages (four seen languages and one unseen language) demonstrate that, although the optimal number of training languages varies across these languages, increasing the number of languages never leads to better performance. These findings suggest that increasing the number of training languages does not necessarily enhance multilingual PRMs. A key reason for this is the fixed amount of training data: as the number of languages grows, the training examples per language decrease. This reduction hinders sufficient training for seen languages and negatively impacts cross-lingual transfer to unseen languages.

5.3 How Much English Data Do We Need for Multilingual PRMs?

While multilingual training with equal number of training examples in each language (PRM-MULTI) generally improves performance compared to English-only training (PRM-CROSS), we observe some exceptions on certain languages, as shown in Table 1. This observation prompts us to investigate how varying the number of English examples can affect the multilingual PRMs.

Setup To explore this, we create data mixtures with varying percentages of English examples ($P\%$), with the remaining $(100 - P)\%$ examples evenly distributed among six languages: German, Spanish, French, Russian, Swahili, and Chinese. Each PRM trained on these mixtures is then evaluated across all 11 languages.

Moderate amount of English data can lead to better multilingual PRMs. As shown in Figure 3, incorporating a small amount of English data into the training mixture can lead to notable performance improvements across languages. Specif-

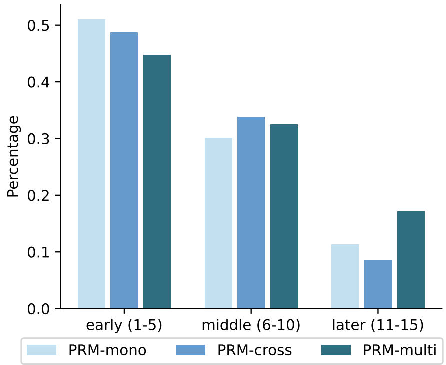

Figure 4: Percentage distribution of the first error positions corresponding to the step in the reasoning on the PRM800K testset.

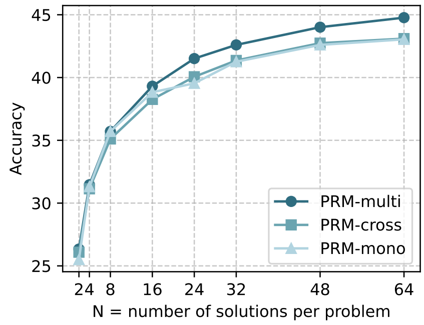

Figure 5: Best-of-N sampling performance of LLAMA-3.1-8B-MATH using different verification strategies across distinct numbers of solutions on `MATH500`.

ically, even as little as 1% of English examples significantly enhances performance, particularly for unseen languages. Interestingly, the majority of performance gains occur when English data constitutes less than 50% of the training mixture. However, when the proportion of English data exceeds 50%, performance begins to decline slightly across languages. Furthermore, training on 70% English data outperforms training solely on English (100%), suggesting that retaining some multilingual data introduces valuable variation and enhances the generalization capacity of multilingual PRMs. These findings indicate that as the proportion of English data increases, the PRMs may not be adequately trained on other seen languages, and unseen languages may benefit less from cross-lingual transfer. This highlights the importance of maintaining diverse and balanced language representation in multilingual training for optimal performance.

6 Analysis

In this section, we present a comprehensive analysis of our multilingual PRM, focusing on five critical aspects: error positions (Section 6.1), number of solutions (Section 6.2), integration of LoRA with PRM (Section 6.3), comparative evaluation with multilingual ORM (see Section 6.4), and implement PPO with multilingual PRM (see Section 6.5).

6.1 Which Steps Are More Prone to Errors?

PRMs provide fine-grained feedback on each intermediate step of a model's chain-of-thought reasoning process. The errors made at intermediate steps can propagate through the reasoning chain, ultimately affecting the final answer. Therefore, in this section, we investigate the earliest errors made by PRMs during the reasoning process, following Zheng et al. (2024).

Setup We select a subset of instances from the PRM800K Russian test set where the final answers made by PRM-MONO, PRM-CROSS, and PRM-MULTI are incorrect. For these instances, we identify the first occurrences of incorrect predictions from these PRMs. We classify the first error positions into three groups: *early* (steps 1 to 5), *middle* (steps 6 to 10), and *later* (steps 11 to 15).

Multilingual PRMs produce fewer errors at early steps. The distribution of the earliest error positions, visualized in Figure 4, reveals a clear distinction between the three PRM configurations. In both PRM-MONO and PRM-CROSS, a significant proportion of errors occurs within the early steps. In contrast, PRM-MULTI demonstrates fewer errors within this range and exhibits a slightly higher number of errors in later steps. These observations suggest that PRM-MULTI may be less prone to error propagation in the reasoning process, enabling it to maintain a more reliable reasoning trajectory. Consequently, PRM-MULTI can effectively achieve better overall performance.

6.2 Do More Candidates Drive Better Performance?

Recent research suggests that providing more candidate solutions can significantly boost the performance of PRM (Lightman et al., 2024; Wang et al., 2024b,a). To investigate whether this trend extends to multilingual settings, we examine the impact of varying the number of candidates on PRM-MONO, PRM-CROSS, and PRM-MULTI.

Setup We conduct experiments on the `MATH500` benchmark using the LLAMA-3.1-8B-MATH gen-

MATH500									
	MISTRAL			LLAMA			DEEPSEEK		
Verifier	μ_{ALL}	μ_{SEEN}	μ_{UNSEEN}	μ_{ALL}	μ_{SEEN}	μ_{UNSEEN}	μ_{ALL}	μ_{SEEN}	μ_{UNSEEN}
BASELINE	22.11	24.34	18.20	22.07	24.34	18.10	26.38	32.48	15.70
SC	29.20	31.80	24.65	30.60	33.31	25.85	44.96	49.29	37.40
ORM	39.54	42.63	**34.25**	40.49	43.14	35.85	50.96	55.54	42.95
PRM-MULTI	**39.55**	**43.11**	33.30	**41.71**	**44.77**	**36.35**	**51.25**	**55.57**	**43.70**
MGSM									
	MISTRAL			LLAMA			DEEPSEEK		
Verifier	μ_{ALL}	μ_{SEEN}	μ_{UNSEEN}	μ_{ALL}	μ_{SEEN}	μ_{UNSEEN}	μ_{ALL}	μ_{SEEN}	μ_{UNSEEN}
BASELINE	49.63	61.65	28.60	56.18	64.23	42.10	52.95	63.02	35.30
SC	56.51	69.37	34.00	63.13	74.57	43.10	70.76	75.37	62.70
ORM	64.84	76.40	44.60	65.20	77.43	43.80	74.44	79.00	66.45
PRM-MULTI	**65.45**	**77.09**	**45.10**	**71.93**	**82.00**	**54.30**	**75.42**	**80.51**	**66.50**

Table 2: Multilingual PRMs' best-of-N (N = 64) sampling performance on MATH500 and MGSM with three generators: METAMATH-MISTRAL-7B, LLAMA-3.1-8B-MATH, and DEEPSEEKMATH-7B-INSTRUCT. We use QWEN2.5-MATH-7B-INSTRUCT to finetune the ORM and PRM-MULTI. μ_{ALL}, μ_{SEEN}, and μ_{UNSEEN} indicate the macro-average of results across all the languages, the seen languages, and the unseen languages, respectively.

erator to compare the performance of multilingual PRM (PRM-MULTI), cross-lingual PRM (PRM-CROSS), and monolingual PRM (PRM-MONO). For each approach, we vary the number of candidates N from 2 to 64. This allows us to assess how the number of candidate solutions influences performance across different PRM strategies in a multilingual context.

Multilingual PRMs yield better performance with more candidate solutions. Figure 5 illustrates that PRM-MULTI consistently outperforms both PRM-CROSS and PRM-MONO, with its advantage growing more pronounced as the number of candidates (N) increases. This finding underscores the scalability of multilingual PRM in diverse linguistic scenarios. Overall, these observations reinforce the conclusion that multilingual PRM not only maintains superior performance but also scales well as more candidates are introduced.

6.3 Are Multilingual PRMs Compatible with Parameter-Efficient Finetuning?

Recent research has demonstrated the effectiveness of parameter-efficient finetuning (PEFT) across a variety of tasks (Houlsby et al., 2019; Li and Liang, 2021). Therefore, we explore whether the PEFT approaches, such as LoRA (Hu et al., 2022), also perform well on multilingual PRMs.

Setup To investigate this question, we employ LoRA on the key, query, and value attention matrices. Specifically, we use a rank of 8 and a dropout rate of 0.05 for both multilingual and cross-lingual PRMs. We train for three epochs with a batch size of 64 and a learning rate of $1e^{-5}$.

LoRA is computationally efficient, but not as good as its fully-finetuning counterpart in multilingual PRMs. Figure 6 demonstrates that fully fine-tuning (FFT) consistently outperforms LoRA in both cross-lingual and multilingual settings. The performance gap becomes larger on the MATH500 dataset, which contains more complex questions compared to MGSM, suggesting that FFT is better suited for tasks requiring deeper reasoning and understanding. These findings align with prior research, which indicates that while PEFT methods may fall short of FFT when tasks demand higher complexity or reasoning capabilities (Biderman et al., 2024). Interestingly, although LoRA-based methods generally lag behind FFT, multilingual LoRA achieves stronger results than cross-lingual LoRA. This highlights the benefits of leveraging multilingual data during parameter-efficient finetuning, as multilingual data likely provides richer data diversity and linguistic coverage.

6.4 Does PRM Surpass ORM in the Multilingual Scenario?

In this section, we explore whether PRM also outperforms Outcome Reward Model (ORM) and self-consistency (SC) (Wang et al., 2022) in multilingual settings.

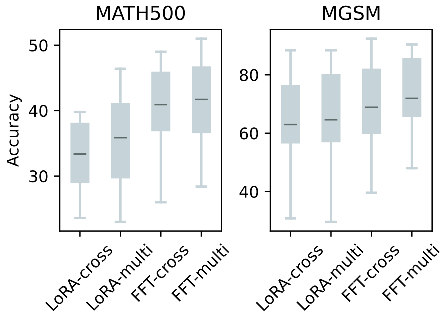

Figure 6: Comparison between parameter-efficient fine-tuning (LoRA) PRM and fully fine-tuning (FFT) PRM with LLAMA-3.1-8B-MATH generator.

	BASELINE	PPO-ORM	PPO-PRM
English	78.40	80.40	**82.40**
German	68.80	64.00	**68.80**
Spanish	72.00	71.20	**76.00**
French	67.60	68.00	**71.60**
Russian	69.60	68.40	**71.20**
Swahili	33.60	38.80	**41.20**
Chinese	59.60	**64.00**	62.80
Japanese	48.80	46.80	**49.20**
Bengali	**45.20**	41.20	40.40
Telugu	17.60	**20.40**	18.00
Thai	**56.80**	51.20	**56.80**
Average	56.18	55.85	**58.04**

Table 3: Zero-shot evaluation on MGSM for LLAMA-3.1-8B-MATH improved via PPO with PRM-MULTI.

Setup Following Lightman et al. (2024); Wang et al. (2024b), we evaluate the performance of PRM-MULTI by comparing it with other *verifier* methods, including: Direct prediction (BASELINE), Self-consistency (majority voting) (SC), and ORM. The accuracy of the best-of-N solution is used as the evaluation metric. Specifically, we train a multilingual ORM using uniform example budgets across seven seen languages. Then we assess the performance of verifiers on seven seen languages as well as on four additional unseen languages.

Multilingual PRM outperforms SC and ORM across all languages and generators. The results presented in Table 2 confirm that PRM consistently achieves higher accuracy on two benchmarks across multiple languages. Specifically, when using the LLAMA-3.1-8B-MATH as the generator, PRM improves average accuracy by +19.64 points on the MATH500 dataset and by +15.75 points on the MGSM dataset in terms of μ_{ALL}, compared to the BASELINE of direct prediction. These substantial gains suggest PRM's potential to enhance reasoning performance in a multilingual setting. Furthermore, PRM also surpasses both SC and ORM. For example, PRM exceeds SC and ORM by margins of up to +8.80 and +6.73 points on MGSM, respectively, when using LLAMA-3.1-8B-MATH as the generator. Additionally, PRM demonstrates performance improvements for both seen and unseen languages. With the DEEPSEEKMATH-7B-INSTRUCT generator on MGSM, PRM achieves respective gains of +17.49 and +31.20 for the seen and unseen language sets, compared to the BASELINE.

6.5 Can Multilingual PRM Enhance LLMs?

We have previously shown that multilingual PRM can bolster model performance under a best-of-N selection framework. In this section, we demonstrate that the multilingual PRM can be used as the reward model for finetuning the LLMs under a reinforcement learning paradigm.

Setup We design experiments to improve LLAMA-3.1-8B-MATH using RL where we adopt the PPO strategy (Schulman et al., 2017) on the MetaMathQA training set (Yu et al., 2023b). We then evaluate the resulting policy models on MGSM using top-1 accuracy in a zero-shot setting. Due to the computational constraints, we only generate one response during the fine-tuning process.

Reinforcement learning with multilingual PRM further improves the performance of LLMs. The results shown in Table 3 indicate that step-by-step PPO with PRM-MULTI (PPO-PRM) consistently outperforms a standard supervised fine-tuned BASELINE and PPO with ORM (PPO-ORM). LLAMA-3.1-8B-MATH with PPO-PRM achieves average boosts of +1.86 and +2.19 across 11 languages, compared to BASELINE and PPO-ORM, respectively. These findings highlight the importance of fine-grained multilingual reward signals. These gains demonstrate that process rewards can refine policy decisions for both reasoning steps and final outputs with reinforcement learning.

7 Conclusion

Our work demonstrates that multilingual PRMs significantly enhance the ability to perform complex, multi-step reasoning tasks in various languages, consistently outperforming both monolingual and cross-lingual counterparts. This conclusion is supported by comprehensive evaluations spanning 11 languages. Furthermore, our findings highlight that performance is sensitive to the number of languages and the volume of English training data. However, it also benefits substantially from more candidate responses and model parameters. These results underscore the importance of diverse language training in providing fine-grained rewards and open up promising avenues for multilingual reasoning.

8 Limitations

While we have demonstrated the effectiveness of multilingual PRMs, our study has not comprehensively explored the wide range of reward optimization methods (Rafailov et al., 2024; Azar et al., 2024), some of which may not benefit from cross-lingual reward model transfer. Nevertheless, best-of-N and PPO, the two techniques leveraged in this paper, are highly representative of current practices, particularly given the consistently strong performance of best-of-N (Gao et al., 2023; Rafailov et al., 2024; Mudgal et al., 2023). Furthermore, while our results show that multilingual PRMs outperform both cross-lingual and monolingual PRMs, our experiments are limited to 11 languages. Extending this approach to a broader set of languages and evaluating its impact across diverse linguistic families is an important avenue for future work.

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	#exam.	max	min	mean
PRM800K trainset	404K	56	1	6.39
Math-Shepherd	445K	30	1	6.23
PRM800K testset	5071	53	1	22.11

Table 4: Dataset statistics of the datasets in this work, including number of examples, maximum, minimum, and average number of steps in the answers.

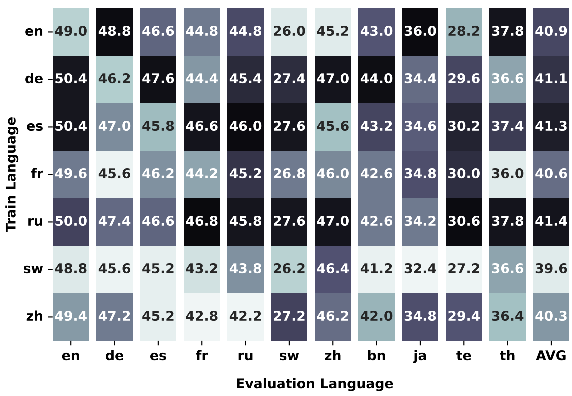

Figure 7: Performance of PRM-MONO trained on seven seen languages and evaluated on all 11 languages based on the MATH500 with LLAMA-3.1-8B-MATH generator.

A Data Statistics

The dataset statistics are summarized in Table 4. These include the total number of examples, as well as the maximum, minimum, and average number of reasoning steps in the answers across all examples.

B Training Details

We train the PRMs by fine-tuning all parameters of QWEN2.5-MATH-7B-INSTRUCT using the AdamW optimizer with a learning rate of 10^{-5} and a batch size of 8. This process is conducted over two epochs on 4 NVIDIA A100 GPUs (80GB). During training, we use a linear learning rate schedule with a warm-up phase that constitutes 10% of the total training steps.

C Cross-lingual Transfer of PRMs

Following Wu et al. (2024b), we assess the performance of cross-lingual PRMs to inspect if language similarity like the script or mutual intelligibility might affect the levels of reasoning verification cross-lingual transfer.

Setup We train PRMs on monolingual versions of the data in German, Spanish, French, Russian, Swahili, and Chinese, and evaluate their transfer to other languages.

No clear signal indicates that language similarity strongly correlates with cross-lingual transfer. We present the cross-lingual transfer results in Figure 7 and observe that there is no clear conclusion regarding the factors that impact cross-lingual transfer. For instance, the PRM trained on Russian data achieves the highest accuracy when evaluating French, Swahili, Chinese, Telugu, and Thai. Notably, these languages neither share the same script nor belong to the same language family as Russian. This observation suggests that linguistic similarity, in terms of script or language family, may not be a decisive factor in cross-lingual transfer. These findings underscore the uncertainty in predicting cross-lingual transfer performance based solely on language similarity. In practice, selecting a diverse set of representative languages for training a multilingual PRM may be a more effective strategy to address this uncertainty and improve performance across a wide range of target languages.

D Breakdown Results of MGSM for PRM-MONO, PRM-CROSS, and PRM-MULTI

We present the breakdown of results for each language on the MGSM in Table 5. The results indicate that the PRM-MULTI consistently outperforms both the PRM-MONO and PRM-CROSS models across languages. This observation aligns with the conclusion drawn in Section 5.1, highlighting the advantages of multilingual training for PRMs.

MGSM	μ_{ALL}	μ_{SEEN}	μ_{UNSEEN}	en	de	es	fr	ru	sw	zh	ja	bn	te	th
							METAMATH-MISTRAL-7B							
PRM-MONO	-	76.0	-	90.8	78.8	81.2	81.6	86.0	36.0	77.6	-	-	-	-
PRM-CROSS	65.2	76.7	**45.2**	**90.8**	**84.4**	85.2	**82.4**	**86.8**	27.2	**80.0**	**76.2**	43.0	7.6	**54.0**
PRM-MULTI	**65.5**	**77.1**	45.1	89.2	83.2	**86.0**	**82.4**	86.4	**33.2**	79.2	75.6	**43.2**	**8.0**	53.6
							LLAMA-3.1-8B-MATH							
PRM-MONO	-	81.7	-	92.4	83.2	88.0	80.4	82.4	**62.4**	**83.2**	-	-	-	-
PRM-CROSS	68.8	79.3	50.6	**92.4**	82.0	88.0	82.0	79.2	50.4	80.8	72.8	39.6	20.8	69.2
PRM-MULTI	**71.9**	**82.0**	**54.3**	90.4	**87.6**	**88.0**	**83.6**	**83.2**	59.6	81.6	**74.0**	**48.0**	**23.6**	**71.6**
							DEEPSEEKMATH-7B-INSTRUCT							
PRM-MONO	-	80.5	-	96.4	**86.4**	90.4	85.2	88.0	**32.0**	85.0	-	-	-	**-**
PRM-CROSS	74.0	79.0	65.1	**96.4**	86.0	91.2	85.6	87.2	18.4	**88.4**	80.0	57.6	51.6	71.2
PRM-MULTI	**75.4**	**80.5**	**66.5**	95.2	84.0	**92.4**	**86.4**	**89.2**	30.0	86.4	**80.8**	**60.8**	**52.4**	**72.0**

Table 5: Different PRMs' best-of-N sampling (N = 64) performance on MGSM with the generator of METAMATH-MISTRAL-7B, LLAMA-3.1-8B-MATH, and DEEPSEEKMATH-7B-INSTRUCT. μ_{ALL}, μ_{SEEN}, and μ_{UNSEEN} indicate the macro-average of results across all the languages, the seen languages, and the unseen languages, respectively.